Pathria 1.7

$$p = \hbar k , \qquad \vec{k} = \frac{\pi}{L}(n_x, n_y, n_z)$$

$$|\vec{k}| = \frac{\pi}{L}|\vec{n}|$$

$$\varepsilon = cp = c\hbar k = \frac{ch}{2\cancel{\pi}}\frac{\cancel{\pi}}{L}|\vec{n}|$$

$$= \frac{ch}{2L}|\vec{n}|$$

$$|\vec{n}| = \frac{2L\varepsilon}{ch} = \frac{2V^{1/3}\varepsilon}{ch}$$

$\Omega(N,V,E) \equiv$ # of ways to arrange N $\vec{n}$ vectors such that the sum of energy is E:

$$\sum_{i=1}^{N} \varepsilon_i = E \iff \sum_{i=1}^{N} |\vec{n}_i| = \frac{2V^{1/3}E}{ch}$$

The explicit dependence of this combinatoric problem on $V^{1/3}E$ gives the isentropic equation of state:

$$V^{1/3}E = \text{constant}.$$

use $P = -\left(\frac{\partial E}{\partial V}\right)_{N,S}$ to get alternative ~~a~~isentropic eq:

$$PV^{4/3} = \text{constant}.$$

$$\Rightarrow \gamma = \frac{4}{3}, \quad \boxed{\frac{c_p}{c_v} = \frac{4}{3}}$$

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